Calculation of the Effective Emissivities of Specular-diffuse Cavities by the Monte Carlo Method

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Abstract. An algorithm of the Monte Carlo method is described which allows evaluation of the effective emissivities of isothermal and nonisothermal specular-diffuse black-body cavities for use in radiometry, photometry and optical pyrometry. The calculation provides estimates of normal spectral effective emissivity for black-body cavities, formed by cone surfaces and a cylinder. It does this for an isothermal cavity and for a cavity having an arbitrary variation of temperature along the cavity length.

1. Introduction

The effective emissivity of a black-body cavity is a quantity of great importance in radiometry, photometry and optical pyrometry, whose accurate estimation is possible only by the application of numerical techniques. The value of the effective emissivity in each individual case depends on the cavity geometry, the optical characteristics of its internal walls, the temperature distribution over the cavity walls and the conditions of observation. The efficiency and advantages of the methods applied are determined both by the accuracy of the results obtained and by their versatility of approach to different cavities.

Local effective emissivity estimates for isothermal and nonisothermal diffuse cavities of the configurations most currently used can be calculated by approximating the integral equations of radiative heat-transfer theory by finite sums [1-4], and by using the iterative procedure and finite-difference approximation of iterated nuclei [5-7]. The method described in [1-4] was successfully applied to an isothermal diffuse cavity with a flat aperture that has a diffuse emitting, but specular reflecting, inner surface [8]. A method for calculating effective emissivities of diffusive cavities was developed [9, 10] that allowed for the vignetting effect for specific reciprocal locations of the cavity and the radiation detector.

Any specular component of reflectivity acquires greater significance with increase in the wavelength. The use of a diffuse model for reflection in the infrared range may lead to uncontrollable errors in calculating the emission characteristics of cavities. For the approximate estimation of the effective emissivity of a cavity having diffuse and specular components of reflectivity, Redgrove and Berry [11] applied a method which takes account of the first and second radiation reflections from the cavity walls and makes a correction term for nonisothermal conditions. With this method, the calculation error is lower the higher the emissivity of the cavity walls.

The Monte Carlo method has long attracted the attention of researchers engaged in radiative heat-transfer studies by its universal character and the possibility it provides to use the mathematical model constructed for detailed comprehensive investigation. A successful application of this approach was reported [12] for calculations of the hemispherical effective emissivity of a diffuse isothermal conical cavity with an emitting or non-emitting baffle. Ono [13] demonstrated the potential diversity of the Monte Carlo method for evaluating the effective emissivities of isothermal specular-diffuse cavities.

Calculations for isothermal black-body cavities are an indispensable step at a certain point in all calculations, but they are not sufficient for high-accuracy measurements.

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The algorithm of the Monte Carlo method described in this paper allows the calculation of directional, including normal (which is essential for a number of applications), effective emissivities of specular-diffuse isothermal and nonisothermal cavities [14]. It has been realized for a cylindrical-conical cavity.

2. Brief Description of the Method

The directional spectral effective emissivity of a nonisothermal cavity is a function of the selected reference temperature T_0 and is defined as

$$\varepsilon_e(\xi, \omega, \lambda, T_\xi, T_0) = \frac{L_e(\xi, \omega, \lambda, T_\xi)}{L_e(\lambda, T_0)} \quad (1)$$

where $L_e(\xi, \omega, \lambda, T_\xi)$ is the spectral radiance concentration of the effective (i.e. the sum of self thermal and reflected) radiation of an infinitesimal element of the cavity wall with coordinate ξ and temperature T in direction ω at wavelength λ; $L_e(\lambda, T_0)$ is the spectral radiance concentration of a black body determined by Planck's law.

Let us write (1) in the form

$$\varepsilon_e(\xi, \omega, \lambda, T_\xi, T_0) = \varepsilon_e(\xi, \omega, \lambda) + \Delta\varepsilon_e(\xi, \omega, \lambda, T_\xi, T_0), \quad (2)$$

where the first term on the right side of the equation is the directional spectral effective emissivity of an isothermal cavity, and the second is a correction term for nonisothermal conditions.

According to Kirchhoff's extended law for isothermal cavities [15],

$$\varepsilon_e(\xi, \omega, \lambda) = 1 - \rho_e(\xi, \omega, \lambda), \quad (3)$$

where $\rho_e(\xi, \omega, \lambda)$ is the directional-hemispherical spectral effective reflectivity for the direction of radiation incident into the cavity which coincides with the direction of the observations ω.

Further analysis is limited to the case of a uniform specular-diffuse model, in which:

(a) optical characteristics are uniform throughout the entire inner surface of the cavity;

(b) radiant emission of the cavity walls is diffuse (i.e. in compliance with Lambert's law), with spectral hemispherical emissivity $\varepsilon(\lambda)$;

(c) the directional-hemispherical spectral reflectivity of the walls,

$$\rho(\lambda) = 1 - \varepsilon(\lambda), \quad (4)$$

is independent of the radiation incidence direction;

(d) spectral reflectivity is a sum of specular $\rho_s(\lambda)$ and diffuse $\rho_d(\lambda)$ components

$$\rho(\lambda) = \rho_s(\lambda) + \rho_d(\lambda); \quad (5)$$

(e) the ratio of specular and diffuse reflectivity components is described by the diffusivity

$$D = \frac{\rho_d(\lambda)}{\rho(\lambda)}, \quad (6)$$

which has a constant value within the wavelength range $\lambda_1 < \lambda < \lambda_2$.

For an isothermal cavity, (3) can be used to calculate the effective reflectivity by the Monte Carlo method; in this case, the incident radiation in direction ω is considered to consist of a rather large number of particles. Initially each particle is assigned a statistical weight of unity: this is multiplied by $\rho(\lambda)$ following every reflection. Specular or diffuse reflection is chosen by means of pseudo-random numbers, as described by Corlett [16]: if for the next in turn pseudo-random number η of the general totality of numbers uniformly distributed within the interval $(0, 1)$ the expression $\eta < D$ holds true, the reflection is considered to be diffuse, otherwise it is specular.

After each diffuse reflection, the statistical weight of the particle is reduced to take account of radiation loss through the cavity aperture: this is done by applying the diffuse angle factor between an element of the wall area at the point of reflection ε and the cavity aperture

$$F(\xi) = \frac{1}{\pi} \int_\Omega \cos\theta_\xi \, d\Omega, \quad (7)$$

where Ω is the solid angle of the aperture from point ξ; θ_ξ is the angle between the normal to the cavity wall at ξ and the direction of the axis of an element of the solid angle $d\Omega$.

Figure 1 shows a cavity formed by the surfaces of a cone and a cylinder. For each of three surfaces forming the cavity (1, a cone bottom; 2, a cylindrical part; 3, a conical diaphragm) the diffuse angle factor can be expressed by applying the superposition principle through angle factors between the cavity aperture and an element of the wall area, respectively either parallel or perpendicular to the aperture plane [17]:

$$F_1(z) = \cos(\chi_1/2) F_\perp[z\tan(\chi_1/2), z_3 - z] + \sin(\chi_1/2) F_\parallel[z\tan(\chi_1/2), z_3 - z] \quad (8)$$

$$F_2(z) = F_\perp(R_1, z_3 - z) \quad (9)$$

$$F_3(z) = \sin(\chi_2/2) F_\perp[R_0 + (z_3 - z)\tan(\chi_2/2), z_3 - z] + \cos(\chi_2/2) F_\parallel[R_0 + (z_3 - z)\tan(\chi_2/2), z_3 - z], \quad (10)$$

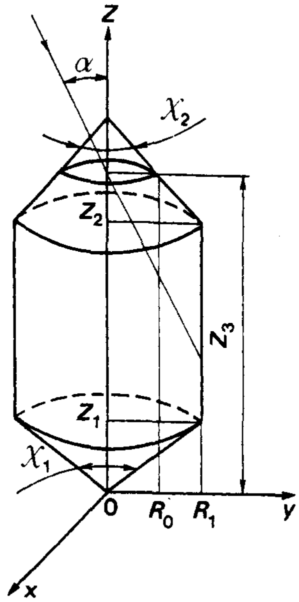

Figure 1. Calculable cavity model.

where

$$F_{\parallel}(r, h)=\frac{1}{2}\{1-[1+(r/h)^2-(R_0/h)^2]/P\} \quad (11)$$

and

$$F_{\perp}(r, h)=\frac{h}{2r}\{[1+(r/h)^2+(R_0/h)^2]/P-1\}. \quad (12)$$

Here r and h are distances from the element of area to the axis and to the plane of the disc (the cavity aperture), respectively, and

$$P=\{[1+(r/h)^2+(R_0/h)^2]^2-4(R_0 r/h^2)^2\}^{1/2}. \quad (13)$$

If, after diffuse reflection, a particle escapes from the cavity through the aperture, the last direction of reflection is ignored and the computation begins for another particle. For a fixed number of particle trajectories N, it is simple to show that

$$\varepsilon(\xi, \omega, \lambda)=1-\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{M}\rho^k(\lambda)F(\xi_k)\prod_{j=1}^{k-1}[1-F(\xi_j)], \quad (14)$$

where M is the maximum of reflections in a trajectory. For specular reflection, $F(\xi)=1$ if a particle escapes from the cavity, otherwise $F(\xi)=0$.

If δ is the admissible component of the calculation error due to the interruption of a particle trajectory and $\varepsilon_{min}=\min\varepsilon(\lambda)$ at $\lambda_1<\lambda<\lambda_2$, the maximum number of reflections in the trajectory can be estimated from the relation [18]

$$M=\mathrm{ent}\left[\frac{\ln(\delta\varepsilon_{min})}{\ln(1-\varepsilon_{min})}\right], \quad (15)$$

where ent$[x]$ is the greatest integer less than M equal to x.

The directional spectral effective emissivity of a nonisothermal cavity can be calculated using the reciprocity theorem [13] which permits substitution of the direction of incident radiation into the cavity for the direction of the observation. Each time a particle is reflected from the wall, the spectral radiance concentration of a black body is calculated at the cavity wall temperature for the reflection point in accordance with Planck's law. The weighted summation of these values along the particle trajectory makes it possible to evaluate the spectral radiance concentration of the effective radiation of point ξ in direction ω. The particle trajectory in the cavity is interrupted when the particle escapes from the cavity through the aperture or when the contribution of some reflection point into the spectral radiance concentration of the effective radiation of point ξ in direction ω becomes negligibly small.

Correction for the nonisothermal case is made using the expression

$$\Delta\varepsilon_e(\xi, \omega, \lambda, T_\xi, T_0)=\frac{\varepsilon(\lambda)}{NL_e(\lambda, T_0)}\sum_{i=1}^{N}\sum_{k=1}^{M}\rho(\lambda)^{k-1}[L_e(\lambda, T_k)-L_e(\lambda, T_0)], \quad (16)$$

where $T_1=T$, T_k, $k=2, 3, \ldots M$ are values of the cavity wall temperature at the reflection points.

A time-saving calculation procedure for the isothermal case involves the assignment, to each particle, of a set of statistical weights and, to the cavity walls, a corresponding set of spectral reflectivities. This method yields the entire spectral relation $\varepsilon(\xi, \omega, \lambda)$ within a single calculation cycle. Similarly, when introducing a correction term for nonisothermal conditions, one may derive the entire spectral relation $\Delta\varepsilon(\xi, \omega, \lambda, T_\xi, T_0)$ at one run.

We are mainly interested in the normal effective emissivities $\varepsilon_{en}(\lambda)$ and $\varepsilon_{en}(\lambda, T_0)$ that are featured when particles are initially directed parallel to the cavity axis and intersections of trajectories with the aperture plane are uniformly distributed over its cross-section. When the aperture image is formed by a wide-angle optical system the angular distribution of effective emissivity $\varepsilon_{a\alpha}(\alpha)$ of a black body, with an angle α between the direction of the observation and the cavity axis (see Figure 1) is of great significance.

The algorithm can easily be modified to treat the case of cavities for which the diffusivity and emissivity vary along the walls. Test calculations for diffusive cavities described by Bedford and Ma [1 – 3] show that an absolute calculation error of the order of 0,0001 can be achieved at $N = 10^3$ to 10^4.

3. Results for an Isothermal Cavity

The developed algorithms and programs have been applied to the design of a black body based on a coaxial heat-pipe with a sodium heat-transfer medium for use as a standard source in infrared spectroradiometry.

The cavity in question (Figure 1) has the following parameters: $R_0 = 5$ mm; $R_1 = 10$ mm; $z_3 = 100$ mm; $\chi_2 = 90°$. The angle χ_1 is the parameter to be optimized using the criterion of maximum normal effective emissivity ε_{en}. The inner cavity surface (made of stainless steel oxidized by heating to 1300 K in air) has emissivity $\varepsilon(\lambda)$ of 0,6 to 0,9 in the 1 µm to 25 µm wavelength range.

Figure 2 demonstrates relations $\varepsilon_{en}(\chi_1)$ for $\varepsilon = 0,6$ and three diffusivity, D, values. If a cavity has a specular component of reflectivity, some of the values

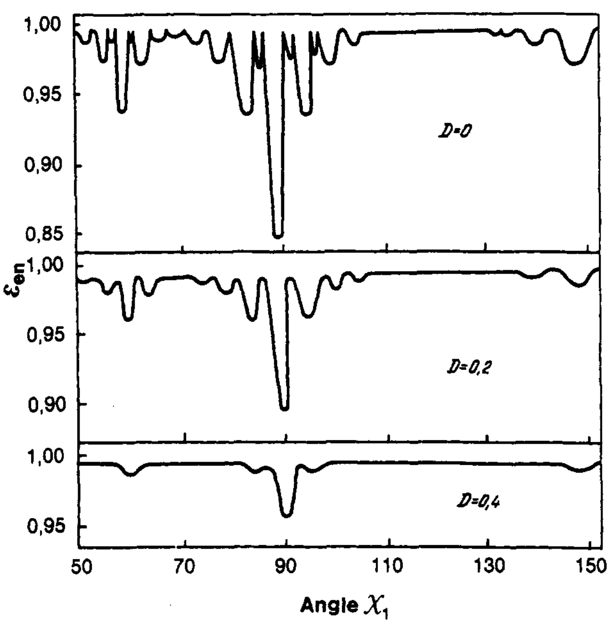

Figure 2. Normal coefficient of emissivity ε_{en} of a cylindrical-conical cavity as a function of the angle χ_1, at the apex of the conical bottom for $\varepsilon = 0,6$ and three diffusivity, D, values.

are characterized by a sharply decreased ε_{en}. Thus, for example, a decrease in ε_{en} near $\chi_1 = 90°$ and $D = 0$ is due to the fact that, in the case of purely specular reflection, a cone with the right apex angle acts as an angular reflector so thermal radiation from the observed area proper, as well as radiation subject to a single reflection, is emitted parallel to the cavity axis. The minima of the $\varepsilon_{en}(\chi_1)$ relation near χ_1 values of 60°, 85°, and 95° are due to effects arising from not more than two reflections. Shallower minima correspond to 3-fold, 4-fold, and more numerous reflections. Within the 115° to 130° range, ε_{en} is close to unity and is independent of the value of D.

For the case of a purely specular cavity and an infinitely narrow beam, the relation between ε_{en} and χ_1 is a stepwise function. By averaging the aperture to correspond to the final width of the beam, this relation is smoothed out. The elevation of ε at the fixed value D leads to shallower minima and an increase in total ε_{en}. If the portion of the diffuse reflectivity component increases while the ε value is kept fixed, the amplitude of the oscillations in the $\varepsilon_{en}(\chi_1)$ relation is reduced, but it does not follow that for all χ_1 values the above increase results in an enhancement of ε_{en}.

At $D = 1$ (a purely diffuse cavity), $\varepsilon_{en}(\chi_1)$ takes the shape of a smooth curve weakly rising at 180°, which is explained by the enhancement of the inner surface area at the fixed cavity depth. Numerical calculations show that the effective emissivity of the black body under development can be increased, if the diffusivity value is different for each of the three surfaces forming the cavity.

If the diffusivity of the bottom surface is $D_1 = 0,2$, the diffusivity of the wall is $D_2 = 0,8$, the diffusivity of the aperture is $D_3 = 0,2$ and the emissivity is $\varepsilon = 0,6$, the effective emissivity is given by $\varepsilon_{en} > 0,9994$. A different degree of diffusivity for the surfaces can be achieved by an appropriate treatment (polishing or, vice versa, making them rougher by threading, etc.).

Figure 3 demonstrates the wavelength dependence of the emissivity of stainless steel (the material used for manufacturing the radiant cavity) and the normal spectral effective emissivity for a cavity having a cone bottom apex angle of 120° and the above diffusivity values for the cavity-forming surfaces. This cavity is detailed in Section 4.

4. Results for a Nonisothermal Cavity

Numerical calculations of normal spectral effective emissivity were made for a cavity having the parameters specified in Section 3, for different distributions of temperature along the cavity. Some of the distributions are listed in Table 1.

Fixed temperatures were set at three points: at the cone bottom apex ($z = 0$), at the cone/cylinder junction ($z = z_1$) and at the cavity edge ($z = z_3$). Temperature values between the indicated points were determined by linear interpolation. In all cases the temperature of the cone bottom apex was taken as the reference temperature T_0. Distributions 1, 2 and 3 of Table 1 are from the model, while distribution 4

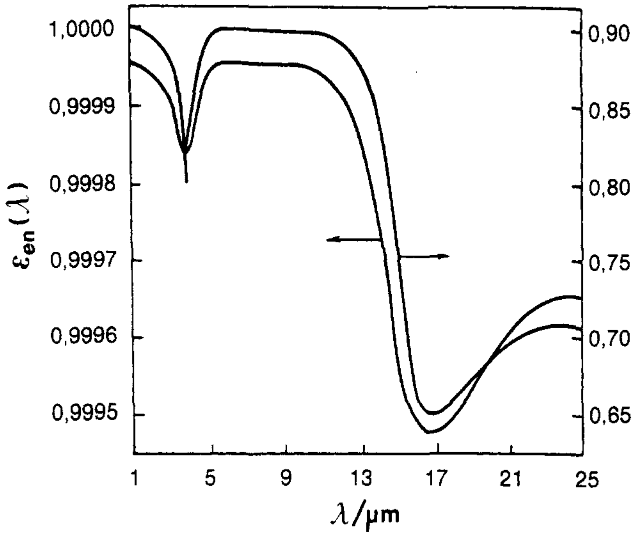

Figure 3. Wavelength dependence of NbC emissivity $\varepsilon(\lambda)$ and normal efficient emissivity $\varepsilon_{en}(\lambda)$ of a black-body cavity made of NbC.

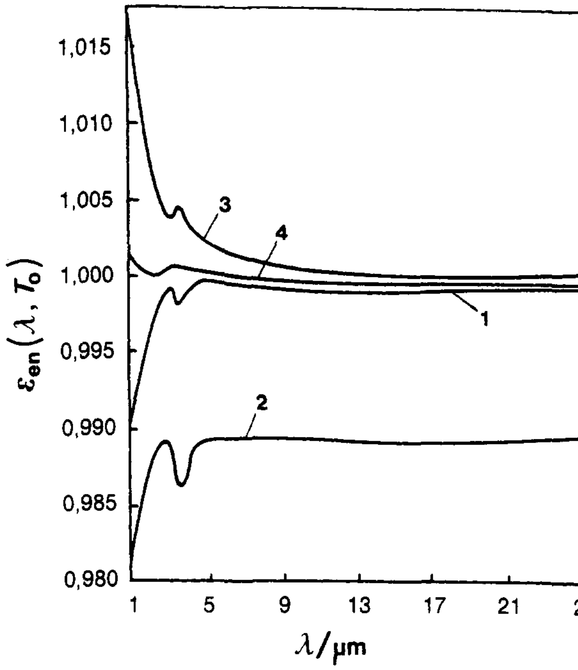

Figure 4. Dependence of normal efficient emissivity $\varepsilon_{en}(\lambda, T_0)$ on temperature distribution along the cavity wall (see Table 1).

Table 1. Set values for distributions N, showing temperature T/K for chosen positions along the z-axis of Figure 1.

N	$z=0$	$z=z_1$	$z=z_3$
1	1000	1000	990
2	1000	998	998
3	1000	1002	1002
4	1000	1000,2	993

is an approximation of the temperature distribution measured with a real black-body cavity.

Figure 4 demonstrates the $\varepsilon(\lambda, T_0)$ relations calculated for the four distributions above. Analysis of the numerical experimental data of Figure 4 suggests the following principal conclusions.

(i) The least difference between the normal spectral effective emissivity of a nonisothermal cavity and that of an isothermal cavity (see Figure 3) is observed in the case of a cone bottom which is isothermal. The greater the dimensions of the isothermal zone near the region of the observation, the lower the absolute value of the correction for nonisothermal conditions.

(ii) For temperature distribution 4, directional spectral effective emissivities were calculated within the range of angles with the cavity axis up to 30°. Over the 1 μm to 25 μm wavelength region, nonuniformity of the temperature distributions does not give a deviation of effective emissivity from that of the isothermal case exceeding 0,001 and the deviation decreases with increasing wavelength.

5. Conclusion

This paper describes a versatile algorithm which uses the Monte Carlo method to calculate the effective emissivities of cavities, taking due account of the relationship of specular and diffuse reflectivity components of the cavity internal walls, the conditions of observation and actual temperature distributions along the radiating surfaces. The algorithm has been realized for a black-body cavity formed by two cone surfaces and a cylinder. By setting the geometric parameters near to the limiting ones the model cavity can be transformed into a cone, a cylinder, a double-cone cavity, and so on. The part of the algorithm which provides for the random wanderings of particles within the cavity is easily modified to suit cavities of other geometries, including those with self-baffled surfaces, asymmetric surfaces, etc. In view of these features, the proposed method of calculation can be considered among the most effective techniques for determining the emission characteristics of black-body cavities.

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Received on 27 February 1991 and in revised form on 7 January 1992.